EFFECT OF INHOMOGENEITIES IN AN EXPLOSIVE ON THE CRITICAL DETONATION DIAMETER

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The effect of inhomogeneities in an explosive on the critical detonation diameter is theoretically analyzed. On the assumption that the inhomogeneities are centers of the reaction behind the detonation front, a formula is obtained for determining the critical diameter as a function of the concentration and size of the inhomogeneities. It is shown that when the inhomogeneities are particles of an inert substance added to the explosive, the dependence of the critical diameter on the mass fraction and size of the inert particles may have a minimum. The results of the theoretical analysis are in qualitative agreement with the experimental data.

Inhomogeneities in an explosive charge may have an important influence on the detonation properties. Thus, as an example, in [1] it was found that introducing inorganic additives ($CaCO_3$, PlO, W, etc.) into an explosive markedly (in some cases by a factor of almost 2) reduces the critical diameter d_*. To explain this effect the authors of [1] proposed the following mechanism of the chemical reaction in a detonation wave propagating along an explosive charge with inert additives. (A substance is considered inert if the heat release resulting from its reaction with the explosive or decompostion takes place behind the Chapman-Jouguet plane.) In the detonation wave (Fig. 1) the chemical reaction is initiated at individual centers or "hot spots" b that develop behind the detonation front a as a result of the interaction of the shock-compressed explosive and the particles of inert substance. Then the reaction is propagated to the adjacent layers of explosive f and ends in the Chapman-Jouguet plane g.

Accordingly, the combustion time τ_c is composed of two parts: an induction period τ_i (the time required for the reaction to develop at the ignition center) and the time required for propagation of the reaction from the ignition centers up to the final completion of combustion τ_r:

$$\tau_c = \tau_i + \tau_r \tag{1}$$

The addition of inert particles may substantially reduce the reaction time τ_c; accordingly, the critical diameter of an explosive containing inert particles may be less than the critical diameter of the pure explosive, since $d_* \sim \tau_c$.

Interesting results relating to the dependence of the critical diameter of a liquid explosive (nitromethane) on the content of powdered Al_2O_3, Al, W, and carbon black were recently obtained by Kurbangalina [2]. It was found that as the mass fraction of inert powder increases, the critical diameter of a nitromethane-Al_2O_3 mixture at first decreases (by several times) and then increases. Moreover, the critical diameter of the mixture also depends on the particle size of the inert powder. At a fixed value of the mass fraction of inert substance there is a particle size at which the critical diameter has a minimum. Kurbangalina attributes this dependence of the critical diameter on the mass fraction and particle size of the inert powder to the important influence of the ignition centers resulting from the interaction of the shock-compressed explosive and the inert particles on the detonation reaction time.

Quantitative estimates of this effect can be obtained on the basis of the detonation model for an explosive charge with inhomogeneities proposed in [1] (Fig. 1).

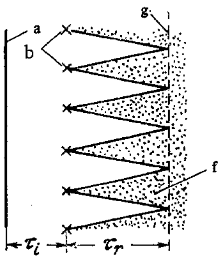

Fig. 1

Thus, we assume that the chemical reaction behind the detonation front is initiated at individual centers—hot spots—resulting from interaction of the shock-compressed explosive and the inert particles, which may take place owing to differences in the physical properties (such as density, hardness, etc.) of the inert substance and the explosive. The critical diameter d_* can be estimated as follows* [3]:

$$d_* \sim D\tau_c \tag{2}$$

Here, D is the detonation velocity, $\tau_c = \tau_i + \tau_r$ is the total reaction (combustion) time (1).

If it is assumed that the particle velocities of the inert substance and the explosive are the same at the instant of completion of the chemical reaction, the Chapman-Jouguet detonation velocity is approximately equal to

$$D \approx D_0 \sqrt{1-\alpha} \tag{3}$$

Here, D_0 is the detonation velocity of the pure explosive, and α is the mass fraction of inert substance.

Let us now calculate the combustion time τ_c (1). We first estimate the induction period τ_i. If we neglect burnup during the time τ_i and hot-spot formation time, the approximate expression for τ_i near the ignition limit has the following form [4]

$$\tau_i \approx \sqrt{2}\pi\tau_q (e\tau_q/\tau_a - 1)^{-1/2} \tag{4}$$

$$\left(\tau_a = \frac{c_p\rho RT^2}{QZE} \exp\frac{E}{RT}, \qquad \tau_q = \frac{2c_p\rho r_*^2}{3\lambda}\right)$$

Here, τ_a is the adiabatic induction period; τ_q is the characteristic heat transfer time; c_p is specific heat; ρ is density; Q is the reaction energy; Z is the pre-exponential factor; R is the gas constant; E is the activation energy; T and r_* are the temperature and size, respectively, of the hot spot; and λ is the thermal conductivity.

Near the ignition limit $\tau_q/\tau_a \approx 1/e$; therefore, by multiplying the numerator and the denominator of the fraction in Eq. (4) by $\sqrt{2}$, we obtain correct to a coefficient of order unity

$$\tau_i \approx \tau_a (1 - \tau_a^2 / e^2\tau_q^2)^{-1/2} \tag{5}$$

Clearly, relation (5) gives the qualitatively correct dependence of τ_i on τ_a/τ_q not only as the ignition limit is approached, when $\tau_a/\tau_q \to e$ and $\tau_i \to \infty$, but also remote from the limit, when $\tau_a/\tau_q \to 0$; in this case $\tau_i \to \tau_a$.

Therefore relation (5) will be perfectly applicable for estimating the induction period τ_i on the entire interval $(0, e)$ of variation of τ_a/τ_q.

The temperature of the hot spot depends both on the detonation velocity and on the physicomechanical properties of the inert substance and the explosive and, clearly, in the first approximation can be determined as follows:

$$T = \frac{\delta D^2}{c_p} \approx \frac{\delta D_0^2 (1-\alpha)}{c_p} \tag{6}$$

The quantity δ depends chiefly on the physicomechanical properties of the inert substance and the explosive.

Using expressions (4)–(6), we can reduce the expression for τ_i to the following form:

$$\tau_i \approx \frac{F}{D} \exp\frac{\theta}{1-\alpha} \left(1 - \frac{\Phi}{r_*^4} \exp\frac{2\theta}{1-\alpha}\right)^{-1/2} \tag{7}$$

$$\theta = \frac{Ec_p}{R\delta D_0^2}, \qquad F = \frac{c_p\rho RTD}{QZE}, \qquad \Phi = \left(\frac{3RT^2\lambda}{2EQZe}\right)^2$$

The reaction propagation time τ_r depends on the distance between hot spots L and the normal burning velocity u. The dependence of u on temperature and pressure usually has a power-law character; therefore in the first approximation for the shock-compressed explosive we may assume that u ~ D. We estimate τ_r as follows:

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*According to Khariton [3] $d_* \approx \tau_c c$, where c is the propagation velocity of the unloading waves in the shock-compressed explosive. In order of magnitude, $c \sim (\Delta p/\Delta\rho)^{1/2}$, where Δp and $\Delta\rho$ are the changes in pressure and density in the unloading wave front. For explosives $\Delta\rho \approx$ const and $\Delta p \sim D^2$, and therefore $c \sim D$ and $d_* \sim \tau_c D$.

$$\tau_r = L / u \approx \beta L / D \qquad (L = N^{-1/3},\ N = n_0 + n) \tag{8}$$

Here, N is the hot spot concentration; n_0 is the concentration of ignition centers resulting from detonation instability; and n is the concentration of inert particles, which we express in terms of their mass fraction:

$$n = \frac{\alpha\rho_0}{{}^4/_3\pi r^3\rho\,[1 - \alpha\,(1 - \rho_0/\rho)]}$$

Here, ρ_0 is the density of the explosive; ρ is the density of the inert substance; and r is the radius of the inert particle. Substituting the expression for L and N into relation (8), we obtain

$$\tau_r \approx \frac{\beta}{D}\,(n_0 + n)^{-1/3} \tag{9}$$

Using relations (1), (2), (7), and (9), for the critical diameter d$_*$ we obtain

$$d_* = \beta\left\{\xi \exp\frac{\theta}{1-\alpha}\left(1 - \frac{\Phi}{r_*^4}\exp\frac{2\theta}{1-\alpha}\right)^{-1/2} + \frac{1}{(n_0+n)^{1/3}}\right\} \qquad \left(\xi = \frac{F}{\beta}\right) \tag{10}$$

In Eq. (10) the coefficients β and θ depend chiefly on the properties of the inert substance and the explosive, and in the first approximation their variation can be neglected. Coefficients ξ and Φ also depend on the detonation velocity, but this dependence can be neglected as compared with the exponential dependence, and therefore ξ and Φ may be assumed constant.

Let us now analyze, with these assumptions, the expressions for the critical diameter d$_*$ (10). First of all, we note the case in which $n_0 \to \infty$ and $r_* \to \infty$. This condition implies that ignition occurs not at individual points, but at all points of the surface at once, i.e., the classical detonation mechanism. In this case the formula for the critical diameter has the same classical form [3]:

$$d_* \sim \exp\frac{\theta}{1-\alpha}$$

and d$_*$ increases with increase in the mass fraction of inert substance (curve 1 in Fig. 2).

More interesting, however, is the case in which the addition of inert particles has an important influence on the properties of the detonation; then the critical diameter depends chiefly on the mass fraction and radius of the inert particles. In this case $n_0 \ll n$ and $r_* \sim r$, and formula (10) assumes the following form:

$$d_* \approx \beta\left\{\xi \exp\frac{\theta}{1-\alpha}\left(1 - \frac{\Phi}{r^4}\exp\frac{2\theta}{1-\alpha}\right)^{-1/2} + \frac{1}{n^{1/3}}\right\} \tag{11}$$

Before turning to an analysis of expression (11), we note that the first term on the right-hand side of (11) is proportional to the combustion propagation time τ_i, while the second is proportional to the combustion propagation time τ_r. Clearly, the dependence of each of these terms on the mass fraction α and radius r of the inert particles is different in character and therefore the dependence of the critical diameter d$_*$ on α and r may have a minimum. In fact, if the hot-spot temperature is sufficiently high, at small values of $\alpha, \tau_i \ll \tau_r$, and the critical diameter is determined by the inert particle concentration:

$$d_* \sim \tau_r \sim \alpha^{-1/3}$$

However, as α increases, the detonation velocity decreases; therefore the induction period τ_i increases, and at a sufficiently large α the opposite inequality $\tau_i \gg \tau_r$ will be satisfied; in this case

$$d_* \sim \tau_i \sim \exp\frac{\theta}{1-\alpha}\left(1 - \frac{\Phi}{r^4}\exp\frac{2\theta}{1-\alpha}\right)^{-1/2}$$

and d$_*$ increases with increase in α.

The graph of the dependence of d$_*$ on α illustrating this case is shown in Fig. 2 (curve 2).

Let us now consider the nature of the dependence of d$_*$ on r (Fig. 3) at a fixed value of α.

If the particle size and hence the size of the hot spots is so small that the ignition condition is not satisfied, i.e., $\tau_a/\tau_q > e$, and therefore the inert particles do not participate in the initiation of the chemical reaction, the critical diameter does not depend on the particle size.

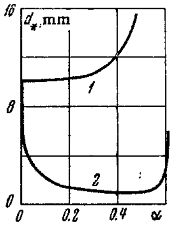

Fig. 2

However, if the particle size is sufficiently large and the ignition condition is satisfied $\tau_a/\tau_q < e$, then at $\tau_i \gg \tau_r$ (near the ignition limit) the critical diameter decreases as the particle radius increases:

$$d_* \sim \tau_i \sim \left(1 - \frac{\Phi}{r^4} \exp \frac{2\theta}{1-\alpha}\right)^{-1/2}$$

With further increase in the particle size an important role begins to be played by the combustion propagation time τ_r; therefore the critical diameter increases with increase in r:

$$d_* \sim \tau_r \sim r$$

Finally, when the particle size becomes so large that the condition

$$n_0 \gg n$$

is satisfied, the critical diameter again becomes independent of r.

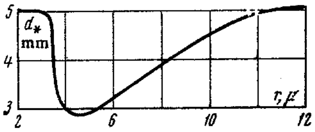

Fig. 3

Thus, it has been shown that the dependence of the critical detonation diameter of a mixture of an explosive and particles of an inert substance on the mass fraction and size of the particles may have a minimum.

In conclusion the author thanks R. Kh. Kurbangalina for formulating and discussing the problem and L. N. Stesik for his useful remarks.

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